SUSY Breaking by Stable non-BPS Walls

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Abstract. A new simple mechanism for SUSY breaking is proposed due to the coexistence of BPS domain walls. It is assumed that our world is on a BPS domain wall and that the other BPS wall breaks the SUSY preserved by our wall. This mechanism requires no messenger fields nor complicated SUSY breaking sector on any of the walls. We obtain an $\mathcal{N}=1$ model in four dimensions which has an exact solution of a stable non-BPS configuration of two walls. We propose that the overlap of the wave functions of the N-G fermion and those of physical fields provides a practical method to evaluate SUSY breaking mass splitting on our wall thanks to a low-energy theorem. This is based on our recent works hep-th/0009023 and hep-th/0107204.

Introduction

Recently it is quite popular to consider models where our world is embedded in a higher dimensional spacetime as a topological defect [1, 2]. At the same time, supersymmetry (SUSY) is one of the most promising ideas to solve the hierarchy problem in unified theories [3]. It has been noted for some years that one of the most important issues for SUSY unified theories is to understand the SUSY breaking in our observable world. Domain walls or other topological defects preserving part of the original SUSY in the fundamental theory are called the BPS states [4] in SUSY theories. Walls have co-dimension one and typically preserve half of the original SUSY, which are called 1/2 BPS states [5].

The new possibility offered by the brane world scenario stimulated studies of SUSY breaking. Recently we have proposed a simple mechanism of SUSY breaking due to the coexistence of different kinds of BPS domain walls and proposed an efficient method to evaluate the SUSY breaking parameters such as the boson-fermion mass-splitting by means of overlap of wave functions involving the Nambu-Goldstone (NG) fermion

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[6]. These points were illustrated by taking a toy model in four dimensions, which allows an exact solution of coexisting walls with a three-dimensional effective theory. Although the first model was only meta-stable, we were able to show approximate evaluation of the overlap allows us to determine the mass-splitting reliably. More recently, we have constructed a stable non-BPS configuration of two walls in an $\mathcal{N}=1$ supersymmetric model in four dimensions to demonstrate our idea of SUSY breaking due to the coexistence of BPS walls. We have also extended our analysis to more realistic case of four-dimensional effective theories and examined the consequences of our mechanism in detail [7].

Our proposal for a SUSY breaking mechanism requires no messenger fields, nor complicated SUSY breaking sector on any of the walls. We assume that our world is on a wall and SUSY is broken only by the coexistence of another wall with some distance from our wall. The NG fermion is localized on the distant wall and its overlap with the wave functions of physical fields on our wall gives the boson-fermion mass-splitting of physical fields on our wall thanks to a low-energy theorem [8]. We have proposed that this overlap provides a practical method to evaluate the mass-splitting in models with SUSY breaking due to the coexisting walls.

Stable non-BPS configuration of two walls

We introduce a simple four-dimensional Wess-Zumino model as follows.

$$\mathcal{L} = \bar{\Phi}\Phi|_{\theta^2\bar{\theta}^2} + W(\Phi)|_{\theta^2} + \text{h.c.}, \qquad W(\Phi) = \frac{\Lambda^3}{g^2}\sin\left(\frac{g}{\Lambda}\Phi\right), \tag{1}$$

where $\Phi = (A, \Psi, F)$ is a chiral superfield. A scale parameter Λ has a mass-dimension one and a coupling constant g is dimensionless, and both of them are real positive. We choose $y = X^2$ as the extra dimension and compactify it on S^1 of radius R. Other coordinates are denoted as x^m $(m = 0, 1, 3)$. The bosonic part of the model is

$$\mathcal{L}_{\text{bosonic}} = -\partial^\mu A^* \partial_\mu A - \frac{\Lambda^4}{g^2}\left|\cos\left(\frac{g}{\Lambda}A\right)\right|^2. \tag{2}$$

The target space of the scalar field A has a topology of a cylinder. This model has two vacua at $A = \pm\pi\Lambda/(2g)$, both lie on the real axis.

Let us first consider the case of the limit $R \to \infty$. In this case, there are two kinds of BPS domain walls in this model. One of them is

$$A_{\text{cl}}^{(1)}(y) = \frac{\Lambda}{g}\left\{2\tan^{-1} e^{\Lambda(y-y_1)} - \frac{\pi}{2}\right\}, \tag{3}$$

which interpolates the vacuum at $A = -\pi\Lambda/(2g)$ to that at $A = \pi\Lambda/(2g)$ as y increases from $y = -\infty$ to $y = \infty$. The other wall is

$$A_{\rm cl}^{(2)}(y) = \frac{\Lambda}{g}\left\{-2\tan^{-1} e^{-\Lambda(y-y_2)} + \frac{3\pi}{2}\right\}, \tag{4}$$

which interpolates the vacuum at $A = \pi\Lambda/(2g)$ to that at $A = 3\pi\Lambda/(2g) = -\pi\Lambda/(2g)$. Here y_1 and y_2 are integration constants and represent the location of the walls along the extra dimension. The four-dimensional supercharge Q_α can be decomposed into two two-component Majorana supercharges $Q_\alpha^{(1)}$ and $Q_\alpha^{(2)}$ which can be regarded as supercharges in three dimensions

$$Q_\alpha = \frac{1}{\sqrt{2}}(Q_\alpha^{(1)} + iQ_\alpha^{(2)}). \tag{5}$$

Each wall breaks a half of the bulk supersymmetry: $Q_\alpha^{(1)}$ is broken by $A_{\rm cl}^{(2)}(y)$, and $Q_\alpha^{(2)}$ by $A_{\rm cl}^{(1)}(y)$. Thus all of the bulk supersymmetry will be broken if these walls coexist.

We will consider such a two-wall system to study the SUSY breaking effects in the low-energy three-dimensional theory on the background. The field configuration of the two walls will wrap around the cylinder in the target space of A as y increases from 0 to $2\pi R$. Such a configuration should be a solution of the equation of motion,

$$\partial^\mu\partial_\mu A + \frac{\Lambda^3}{g}\sin\left(\frac{g}{\Lambda}A^*\right)\cos\left(\frac{g}{\Lambda}A\right) = 0. \tag{6}$$

We find that a general real static solution of Eq.(6) that depends only on y is

$$A_{\rm cl}(y) = \frac{\Lambda}{g}{\rm am}\left(\frac{\Lambda}{k}(y-y_0), k\right), \tag{7}$$

where k and y_0 are real parameters and the function $\phi = {\rm am}(u,k)$ denotes the amplitude function, which is defined as an inverse function of

$$u(\varphi) = \int_0^\varphi \frac{{\rm d}\theta}{\sqrt{1-k^2\sin^2\theta}}. \tag{8}$$

If $k > 1$, it becomes a periodic function with the period $4K(1/k)/\Lambda$, where the function $K(k)$ is the complete elliptic integral of the first kind. If $k < 1$, the solution $A_{\rm cl}(y)$ is a monotonically increasing function with

$$A_{\rm cl}\left(y + \frac{4kK(k)}{\Lambda}\right) = A_{\rm cl}(y) + 2\pi\frac{\Lambda}{g}. \tag{9}$$

Since the field A is an angular variable $A = A + 2\pi\Lambda/g$, we can choose the compactified radius $2\pi R = 4kK(k)/\Lambda$ so that the classical field configuration $A_{\rm cl}(y)$ contains two walls and becomes periodic modulo $2\pi\Lambda/g$. We shall take $y_0 = 0$ to locate one of the walls at $y = 0$. Then we find that the other wall is located at the anti-podal point $y = \pi R$ of the compactified circle.

In the limit of $R \to \infty$, *i.e.*, $k \to 1$, $A_{\rm cl}(y)$ approaches to the BPS configuration $A^{(1)}_{\rm cl}(y)$ with $y_1 = 0$ near $y = 0$, which preserves $Q^{(1)}$, and to $A^{(2)}_{\rm cl}(y)$ with $y_2 = \pi R$ near $y = \pi R$, which preserves $Q^{(2)}$. We will refer to the wall at $y = 0$ as "our wall" and the wall at $y = \pi R$ as "the other wall".

The fluctuation mode expansion

Let us consider the fluctuation fields around the background $A_{\rm cl}(y)$,

$$\begin{aligned} A(X) &= A_{\rm cl}(y) + \frac{1}{\sqrt{2}}(A_{\rm R}(X) + iA_{\rm I}(X)), \\ \Psi_\alpha(X) &= \frac{1}{\sqrt{2}}(\Psi^{(1)}_\alpha(X) + i\Psi^{(2)}_\alpha(X)). \end{aligned} \tag{10}$$

To expand them in modes, we define the mode functions as solutions of equations:

$$\begin{aligned} \left\{-\partial_y^2 - \Lambda^2 \cos\left(\frac{2g}{\Lambda}A_{\rm cl}(y)\right)\right\} b_{{\rm R},p}(y) &= m^2_{{\rm R},p} b_{{\rm R},p}(y), \\ \{-\partial_y^2 + \Lambda^2\} b_{{\rm I},p}(y) &= m^2_{{\rm I},p}(y) b_{{\rm I},p}(y), \end{aligned} \tag{11}$$

$$\begin{aligned} \left\{-\partial_y - \Lambda \sin\left(\frac{g}{\Lambda}A_{\rm cl}(y)\right)\right\} f^{(1)}_p(y) &= m_p f^{(2)}_p(y), \\ \left\{\partial_y - \Lambda \sin\left(\frac{g}{\Lambda}A_{\rm cl}(y)\right)\right\} f^{(2)}_p(y) &= m_p f^{(1)}_p(y). \end{aligned} \tag{12}$$

The four-dimensional fluctuation fields can be expanded as

$$A_{\rm R}(X) = \sum_p b_{{\rm R},p}(y) a_{{\rm R},p}(x), \quad A_{\rm I}(X) = \sum_p b_{{\rm I},p}(y) a_{{\rm I},p}(x), \tag{13}$$

$$\Psi^{(1)}(X) = \sum_p f^{(1)}_p(y)\psi^{(1)}_p(x), \quad \Psi^{(2)}(X) = \sum_p f^{(2)}_p(y)\psi^{(2)}_p(x). \tag{14}$$

As a consequence of the linearized equation of motion, the coefficient $a_{{\rm R},p}(x)$ and $a_{{\rm I},p}(x)$ are scalar fields in three-dimensional effective theory with masses $m_{{\rm R},p}$ and $m_{{\rm I},p}$, and $\psi^{(1)}_p(x)$ and $\psi^{(2)}_p(x)$ are three-dimensional spinor fields with masses m_p, respectively.

Exact mode functions and mass-eigenvalues are known for several light modes of $b_{{\rm R},p}(y)$,

$$\begin{aligned} b_{{\rm R},0}(y) &= C_{{\rm R},0}{\rm dn}\left(\frac{\Lambda y}{k}, k\right), \quad m^2_{{\rm R},0} = 0, \\ b_{{\rm R},1}(y) &= C_{{\rm R},1}{\rm cn}\left(\frac{\Lambda y}{k}, k\right), \quad m^2_{{\rm R},1} = \frac{1-k^2}{k^2}\Lambda^2, \\ b_{{\rm R},2}(y) &= C_{{\rm R},2}{\rm sn}\left(\frac{\Lambda y}{k}, k\right), \quad m^2_{{\rm R},2} = \frac{\Lambda^2}{k^2}, \end{aligned} \tag{15}$$

where functions $\mathrm{dn}(u,k)$, $\mathrm{cn}(u,k)$, $\mathrm{sn}(u,k)$ are the Jacobi's elliptic functions and $C_{\mathrm{R},p}$ are normalization factors. For $b_{\mathrm{I},p}(y)$, we can find all the eigenmodes

$$b_{\mathrm{I},p}(y) = \frac{1}{\sqrt{2\pi R}} \mathrm{e}^{i\frac{p}{R}y}, \quad m_{\mathrm{I},p}^2 = \Lambda^2 + \frac{p^2}{R^2}, \quad (p \in \mathbf{Z}). \tag{16}$$

The massless field $a_{\mathrm{R},0}(x)$ is the Nambu-Goldstone (NG) boson for the breaking of the translational invariance in the extra dimension. The first massive field $a_{\mathrm{R},1}(x)$ corresponds to the oscillation of the background wall around the anti-podal equilibrium point and hence becomes massless in the limit of $R \to \infty$. All the other bosonic fields remain massive in that limit.

For fermions, only zero modes are known explicitly,

$$f_0^{(1)}(y) = C_0 \left\{ \mathrm{dn}\left(\frac{\Lambda y}{k}, k\right) + k\mathrm{cn}\left(\frac{\Lambda y}{k}, k\right) \right\}, \quad f_0^{(2)}(y) = C_0 \left\{ \mathrm{dn}\left(\frac{\Lambda y}{k}, k\right) - k\mathrm{cn}\left(\frac{\Lambda y}{k}, k\right) \right\}, \tag{17}$$

where C_0 is a normalization factor. These fermionic zero modes are the NG fermions for the breaking of $Q^{(1)}$-SUSY and $Q^{(2)}$-SUSY, respectively.

Thus there are four fields which are massless or become massless in the limit of $R \to \infty$: $a_{\mathrm{R},0}(x)$, $a_{\mathrm{R},1}(x)$, $\psi_0^{(1)}(x)$ and $\psi_0^{(2)}(x)$. Other fields are heavier and have masses of the order of Λ.

In the following discussion, we will concentrate ourselves on the breaking of the $Q^{(1)}$-SUSY, which is approximately preserved by our wall at $y=0$. So we call the field $\psi_0^{(2)}(x)$ the NG fermion in the rest of the paper.

Three-dimensional effective Lagrangian

We can obtain a three-dimensional effective Lagrangian by substituting the mode-expanded fields Eq.(13) and Eq.(14) into the Lagrangian (1), and carrying out an integration over y

$$\begin{aligned} \mathcal{L}^{(3)} = & -V_0 - \frac{1}{2}\partial^m a_{\mathrm{R},0}\partial_m a_{\mathrm{R},0} - \frac{1}{2}\partial^m a_{\mathrm{R},1}\partial_m a_{\mathrm{R},1} - \frac{i}{2}\psi_0^{(1)}\partial\!\!\!/\psi_0^{(1)} - \frac{i}{2}\psi_0^{(2)}\partial\!\!\!/\psi_0^{(2)} \\ & -\frac{1}{2}m_{\mathrm{R},1}^2 a_{\mathrm{R},1}^2 + g_{\mathrm{eff}} a_{\mathrm{R},1}\psi_0^{(1)}\psi_0^{(2)} + \cdots, \end{aligned} \tag{18}$$

where $\partial\!\!\!/ \equiv \gamma_{(3)}^m \partial_m$ and an abbreviation denotes terms involving heavier fields and higher-dimensional terms. Here γ-matrices in three dimensions are defined by $\left(\gamma_{(3)}^m\right) \equiv (-\sigma^2, i\sigma^3, -i\sigma^1)$. The vacuum energy V_0 is given by the energy density of the background and thus

$$V_0 \equiv \int_{-\pi R}^{\pi R} \mathrm{d}y \left\{ (\partial_y A_{\mathrm{cl}}) + \frac{\Lambda^4}{g^2}\cos^2\left(\frac{g}{\Lambda}A_{\mathrm{cl}}\right) \right\} = \frac{\Lambda^3}{g^2 k}\int_{-2K(k)}^{2K(k)} \mathrm{d}u \left\{ (1+k^2) - 2k^2\mathrm{sn}^2(u,k) \right\}, \tag{19}$$

and the effective Yukawa coupling $g_{\rm eff}$ is

$$g_{\rm eff} \equiv \frac{g}{\sqrt{2}} \int_{-\pi R}^{\pi R} {\rm d}y \, \cos\left(\frac{g}{\Lambda} A_{\rm cl}(y)\right) b_{{\rm R},1}(y) f_0^{(1)}(y) f_0^{(2)}(y) = \frac{g}{\sqrt{2}} \frac{C_0^2}{C_{{\rm R},1}} (1-k^2). \tag{20}$$

In the limit of $R \to \infty$, the parameters $m_{{\rm R},1}$ and $g_{\rm eff}$ vanish and thus we can redefine the bosonic massless fields as

$$\begin{pmatrix} a_0^{(1)} \\ a_0^{(2)} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} a_{{\rm R},0} \\ a_{{\rm R},1} \end{pmatrix}. \tag{21}$$

In this case, the fields $a_0^{(1)}(x)$ and $\psi_0^{(1)}(x)$ form a supermultiplet for $Q^{(1)}$-SUSY and their mode functions are both localized on our wall. The fields $a_0^{(2)}(x)$ and $\psi_0^{(2)}(x)$ are singlets for $Q^{(1)}$-SUSY and are localized on the other wall.[1]

When the distance between the walls πR is finite, $Q^{(1)}$-SUSY is broken and the mass-splittings between bosonic and fermionic modes are induced. The mass squared $m_{{\rm R},1}^2$ in Eq.(18) corresponds to the difference of the mass squared Δm^2 between $a_0^{(1)}(x)$ and $\psi_0^{(1)}(x)$ since the fermionic mode $\psi_0^{(1)}(x)$ is massless. Besides the mass terms, we can read off the SUSY breaking effects from the Yukawa couplings like $g_{\rm eff}$.

We have noticed in Ref.[6] that these two SUSY breaking parameters, $m_{{\rm R},1}$ and $g_{\rm eff}$, are related by the low-energy theorem associated with the spontaneous breaking of SUSY. In our case, the low-energy theorem becomes

$$\frac{g_{\rm eff}}{m_{{\rm R},1}^2} = \frac{1}{2f}. \tag{22}$$

where f is an order parameter of the SUSY breaking, and it is given by the square root of the vacuum (classical background) energy density V_0 in Eq.(19). Since the superpartner of the fermionic field $\psi_0^{(1)}(x)$ is a mixture of mass-eigenstates, we had to take into account the mixing Eq.(21).

The mass-splitting decays exponentially as the wall distance increases. This is one of the characteristic features of our SUSY breaking mechanism. This fact can be easily understood by remembering the profile of each modes. Note that the mass-splitting $\Delta m^2 (= m_{{\rm R},1}^2)$ is proportional to the effective Yukawa coupling constant $g_{\rm eff}$, which is represented by an overlap integral of the mode functions. Here the mode functions of the fermionic field $\psi_0^{(1)}(x)$ and its superpartner are both localized on our wall, and that of the NG fermion $\psi_0^{(2)}(x)$ is localized on the other wall. Therefore the mass-splitting becomes exponentially small when the distance between the walls increases, because of exponentially dumping tails of the mode functions.

[1]The modes $a_0^{(2)}(x)$ and $\psi_0^{(2)}(x)$ form a supermultiplet for $Q^{(2)}$-SUSY.

We have worked out in Ref. [7] how various soft SUSY breaking terms can arise in our framework. Phenomenological implications have also been briefly discussed.

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